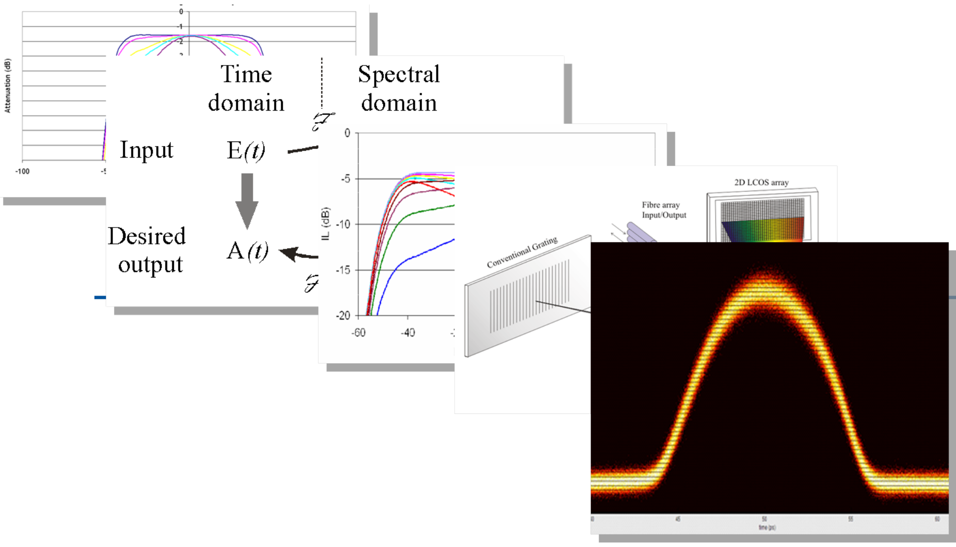

Filter Bandwidth Definition of the WaveShaper S-series Programmable Optical Processor

Finisar®

Section 1: Introduction

The WaveShaper family of Programmable Optical Processors allow creation of user-customized filter profiles over the C- or L- band, providing a flexible tool for industrial or research laboratories. Bandpass filter profiles can be created with a bandwidth ranging from 10 GHz to 5 THz, in 1 GHz steps. However, upon measuring the spectral response of the filter, users can be confused by the bandwidth definition, which conforms to neither a full-width half-maximum (FWHM) nor a 1/e measurement. Furthermore, an inspection of the shape of a nominally flat-top filter shows that there is a finite slope to the roll-off edge of the filter. This can limit the adjacent channel extinction, particularly for very narrow filter bandwidths. What does the specified bandwidth refer to - and is it constant for all devices and channels?

This white paper aims to provide the reader with an understanding of how the Finisar WaveShaper Programmable Optical Processor can generate spectral filters that are defined as having a bandwidth, *B*. First, a theoretical treatment of the filter shape is presented, with a prediction of how the filter bandwidth should be specified. Second, experimental results are shown which confirm the theoretical analysis. Finally, these findings are summarized and clearly stated for the reader.

Section 2: Theory

In this section, an expression for the default, flat-top filter shape produced by the WaveShaper family of Programmable Optical Filters is presented, along with an analysis to appropriately define the filter bandwidth specification. It should be noted that this analysis is derived for a simple flat-top filter response with no additional amplitude or phase shaping.

2.1 Prediction of filter shape

An ideal optical filter of bandwidth, *B*, should look like a rectangular function, from *-B/2* to *B/2*. That is, an ideal channel shape can be expressed as

$$R(f) = \begin{cases} 1, & -B/2 \leq f \geq B/2 \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

This rectangular function is generated on the liquid crystal-on-silicon (LCOS) chip, which sits in the Fourier plane of a *4-f* imaging system. Propagation through the optical system can be reduced to an optical transfer function that is composed of a relatively narrow Gaussian spectrum, given by

$$L(f) = Ae^{-(f-f_o)^2/2\sigma^2}, \quad (2)$$

where *A* is the spectrum amplitude, f_o is the filter center frequency and σ^2 is the spectral variance of the Gaussian spectrum. The spectral variance is related, through the standard deviation, to the 3 dB bandwidth of a Gaussian spectrum by the following relation:

$$\sigma = \frac{BW_{3dB}}{2\sqrt{2\ln 2}}. \quad (3)$$

The bandwidth of the optical transfer function Gaussian is roughly 10 GHz, which suggests that the standard deviation is equal to 4.2466 GHz.

The filter shape generated is then given by the convolution of (1) and (2), or

$$\begin{aligned} S(f) &= R(f) * L(f) \\ &= \int_{-\infty}^{+\infty} R(f')\, L(f'-f)\, df'. \end{aligned} \tag{4}$$

Substituting (1) and (2) into (4), and setting $f_o = 0$, $A = 1$, the integral becomes

$$S(f) = \int_{-B/2}^{B/2} e^{-(f'-f)^2/2\sigma^2} df'. \tag{5}$$

In order to evaluate this integral, a variable substitution is required. Redefining the variable *u* to be equal to

$$u = \frac{f'-f}{\sqrt{2}\sigma}, \tag{6}$$

allows (5) to be rewritten as

$$S(u) = 2\sqrt{2}\sigma \int_{u_1}^{u_2} e^{-u^2} du, \tag{7}$$

where

$$\begin{aligned} u_1 &= \frac{-B/2 - f}{\sqrt{2}\sigma} \\ u_2 &= \frac{B/2 - f}{\sqrt{2}\sigma}. \end{aligned} \tag{8}$$

Evaluation of (7) requires use of the error function, which is commonly defined as

$$\frac{\sqrt{\pi}}{2}\mathrm{erf}(a) = \int_0^a e^{-t^2} dt. \tag{9}$$

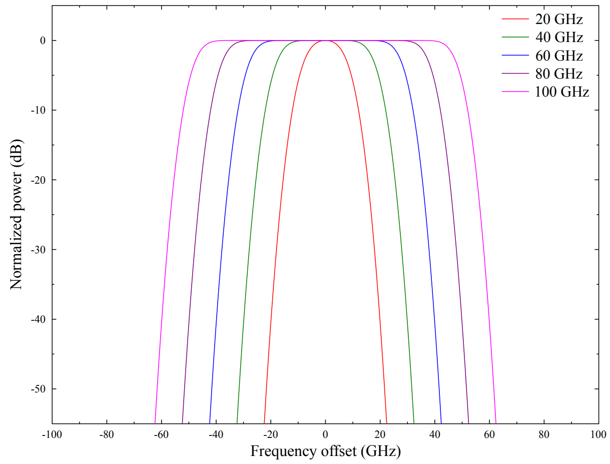

Figure 1 *The filter shape, S(f), calculated for different filter bandwidths, B.*

Using (9), the integral in (7) can be evaluated, resulting in the final form of the channel shape generated by the WaveShaper Programmable Optical Processor, given by

$$S(f) = \sigma\sqrt{2\pi}\left[\operatorname{erf}\left(\frac{B/2 - f}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{-B/2 - f}{\sqrt{2}\sigma}\right)\right]. \qquad (10)$$

Examples of this filter shape are shown in Figure 1, *S(f)* calculated for several different values of *B* and an optical transfer function bandwidth of 10 GHz. The net effect of convolution with a Gaussian is to smooth out the sharp features of the rectangular function, which makes the output spectrum narrower than the original rectangular profile at the top and wider at the bottom, necessitating the existence of a point in the spectrum that crosses the rectangular profile boundary. This “crossover point” marks the relative power level at which the actual filter shape has a bandwidth, *B*.

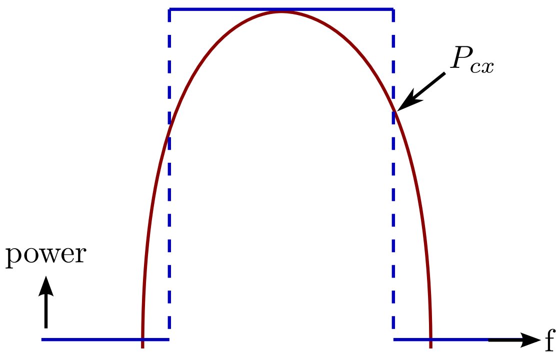

Figure 2 *Identifying the location of the crossover point, where the filter shape (red) crosses the ideal rectangular window (blue.*

2.2 Defining the channel bandwidth specification

In the above analysis, the desired filter profile is a rectangular function with bandwidth, *B*. The actual filter shape generated, however, is narrower at the top part of the channel, and spills out of the rectangular window at the bottom, even though the channel is labeled as a bandpass filter with bandwidth, *B*.

To clearly define the bandwidth of the actual filter shape, it is instructional to look at the power level at which *S(f)* crosses the rectangular function boundary. This is illustrated in Figure 2, where the crossover point, is identified as the point where the generated filter shape crosses the ideal rectangular window.

If the crossover point, P_{cx}, occurs at a constant power level, this power level can be used to strictly define the bandwidth of the filter response.

The power ratio at which P_{cx} occurs, for a given filter bandwidth, *B*, is given by

$$P_{cx} = \frac{S(-{}^{B}\!/_{2})}{S(0)}, \tag{11}$$

where *S(f)* was defined in the previous section. Using (10), this can be evaluated and simplified to

$$P_{cx} = \frac{\mathrm{erf}\,(\gamma)}{2\ \mathrm{erf}\,({}^{\gamma}\!/_{2})}, \tag{12}$$

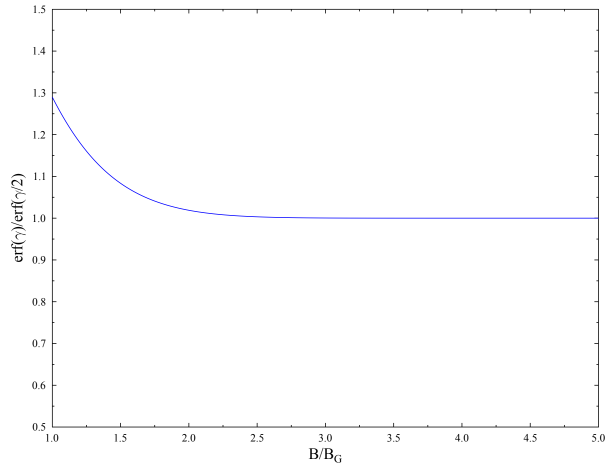

Figure 3 *Error function ratio trend as the ratio of filter bandwidth to optical transfer function bandwidth increases.*

where

$$\gamma = \frac{B}{\sqrt{2}\sigma}. \tag{13}$$

Previously, a relation between the 3 dB bandwidth of the Gaussian filter, B_G, and σ was stated in (3), which is used here to express γ as

$$\gamma = 2\sqrt{\ln 2}\,\frac{B}{B_G}. \tag{14}$$

Hence, an expression for the crossover point, P_{cx}, is found in terms of the ratio between the filter bandwidth, *B*, and the innate optical transfer function bandwidth, B_G. It should be noted that this only holds for a Gaussian spectrum.

Further conclusions can be drawn by considering the nature of the error function, which tends to ½ as γ increases. This is illustrated in Figure 3 where the ratio of error functions is calculated as B/B_G is increased.

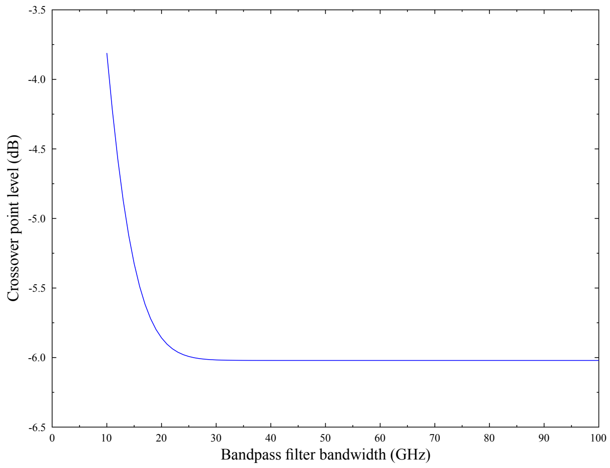

Figure 4 *Location of crossover point with respect to actual filter bandwidth.*

Clearly, for large ratios of B/B_G, the ratio of error functions tend to one; conservatively speaking, it appears safe to conclude that when the desired filter response is at least 3 times larger than the optical transfer function bandwidth, the crossover point occurs at

$$\text{for } B \gg B_G, \qquad P_{cx} \approx \tfrac{1}{2}. \tag{15}$$

This translates to, on a decibel scale, a level that is -6.02 dB down from the filter peak. This is illustrated in Figure 4, which plots the expression given in(12) as a function of the actual filter bandwidth. For values of B that are at least three times larger than B_G, it is evident that the crossover point is located at, roughly, -6 dB from the filter peak.

To reiterate, when a WaveShaper Programmable Optical Processor is set to produce a rectangular function of bandwidth B, which is at least three times larger than the optical transfer function bandwidth, the output filter response will have, roughly, a 6 dB bandwidth of B. If the bandwidth B is less than three times the optical transfer function bandwidth, the bandwidth is defined at the crossover point indicated in Figure 4.

Section 3: Experimental Results

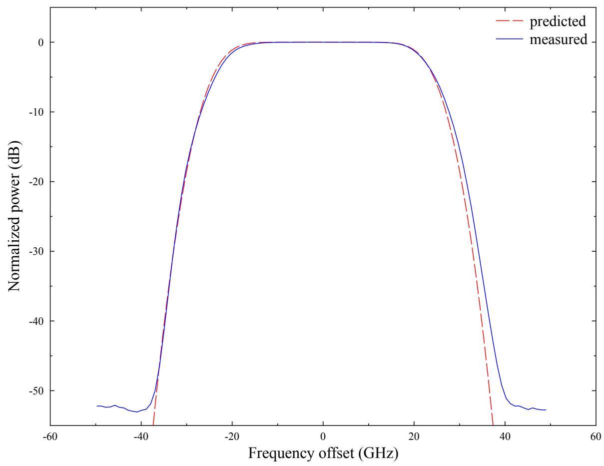

Figure 5 *Comparison between predicted and measured channel shape for a 50 GHz filter response.*

In order to compare the presented theory to measured results, production data from a range of WaveShaper Programmable Optical Processors were extracted and analyzed.

One 50 GHz channel, from a WaveShaper 1000S, was taken to compare to the predicted filter shape shown in (10). Measured results are compared to this predicted channel shape in Figure 5. The spectra match reasonably well, with the exception of a small broadening on one side of the experimental data, which is attributed to the asymmetric beam shape through the device.

To compare the output of a multiport device with filter profiles sent to unique ports, a WaveShaper 4000S was set to output adjacent 10 GHz bandpass filters to each port. The measured data is shown in Figure 6 compared to predicted channel shapes, as derived in the previous section. It should be noted that an optical transfer function bandwidth of 12 GHz was used in these calculations, which takes into account the resolution bandwidth of the optical spectrum analyzer.

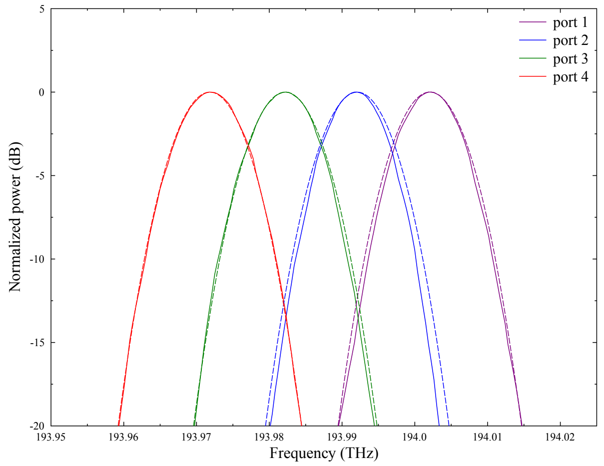

Figure 6 *Four port operation with 10 GHz channel spacing comparing (solid) measured and (dashed) predicted data.*

The predicted model produces an excellent fit compared to the nominal 10 GHz bandpass filter responses, with a slight broadening of the measured data from port 3. From (12), the predicted crossover point of a 10 GHz bandpass filter, using an optical transfer function bandwidth of 12 GHz, is -3.03 dB. From the measured spectra in Figure 6, the average crossover point occurred at -3.38 dB.

Similarly, the behaviour of 50 GHz channels can be predicted with the presented models. A WaveShaper 4000S was therefore set to output adjacent 50 GHz channels to unique ports, and the measured data was compared with predicted spectra, with the results shown in Figure 7.

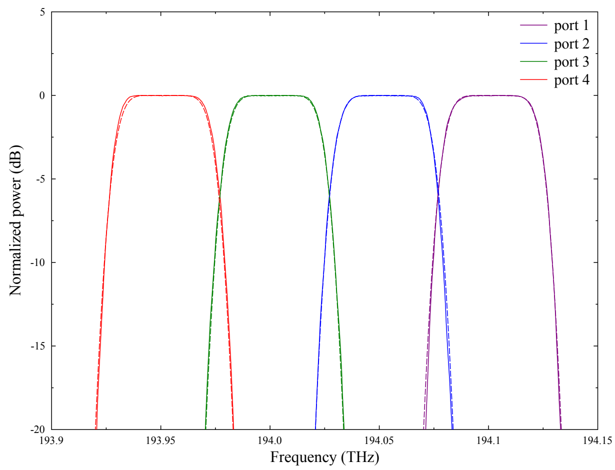

Figure 7 *Four port operation with 50 GHz channel spacing, comparing (solid) measured and (dashed) predicted data.*

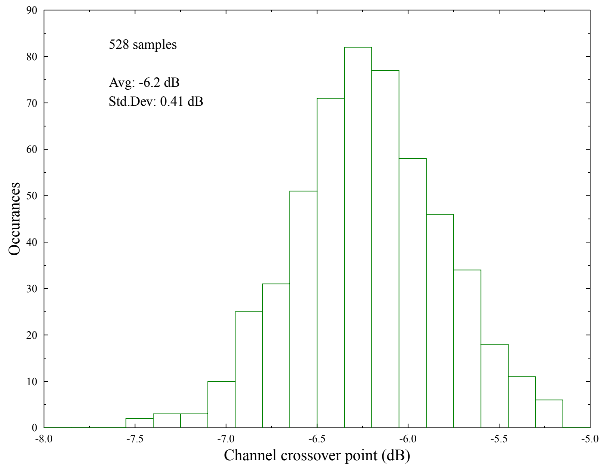

Figure 8 *Survey of crossover points for six interleaved devices, resulting in 528 samples.*

As the generated filter bandwidth of 50 GHz was much larger than the optical transfer function, the experimental data shows an even better fit to theory. The crossover point of the measured spectra occurred at an average of -5.98 dB; in comparison, the predicted spectra have crossover points at -6.02 dB.

To draw a conclusion about the location of the crossover point of a WaveShaper Programmable Optical Processor filter response, a relatively large sample size was required. Six WaveShaper processors were measured; each device was set to produce 50 GHz interleaver patterns, odd and even, sliced into distinct channels, and the crossover point was measured for each slice. This resulted in a total of 528 measurements, giving an appropriate sample size to determine statistical data about the location of the crossover point of a WaveShaper bandpass filter.

The crossover point analysis is shown in Figure 8, tabulated in histogram format. For all samples, the mean crossover point location occurs at -6.2 dB, with a standard deviation of 0.41 dB; the standard deviation is acceptable considering the resolution of the measurement data.

The simple mathematical model presented in this white paper gives accurate agreement with measured spectral data, allowing accurate definition of the crossover point, confirmed by statistical data.

Section 4: Conclusion

This paper has presented a theoretical analysis of the bandpass filter shape generated by the WaveShaper family of Programmable Optical Processors, given as the convolution between a rectangular profile and a narrow Gaussian spectrum. Good matching between theory and experimental data was found, confirming the accuracy of this approach.

Additionally, it was predicted that the crossover point between the output filter response and the initial rectangular profile of bandwidth *B* would occur at roughly -6 dB, if the width of the rectangular profile was greater than 30 GHz. For filter profiles narrower than 30 GHz, the crossover point was found to be given by

$$P_{cx} = \frac{\mathrm{erf}\,(\gamma)}{2\ \mathrm{erf}\,(\gamma/2)}, \tag{16}$$

where

$$\gamma = 2\sqrt{\ln 2}\ \frac{B}{B_G}. \tag{17}$$

This was confirmed by experimental data; over 528 measurements for production devices were used to build a statistical evaluation of the location of the crossover point of 50 GHz bandpass filter responses. The mean crossover point of the sample set was found to be -6.2 dB, with a standard deviation of 0.41 dB.

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